MT 3700 - Differential Equations
WS 1

Predator-Prey Systems

Introduction:

No species lives in isolation, and the interactions among species give some of the most interesting models to study. Consider a simple predator-prey system of differential equations where one species "eats" another. This model will have two dependent variables since the number of predators and the number of prey are both functions of time. Since both predators and prey begin with "*p*", we call the prey "rabbit" (*R*)" and the predators "foxes (*F*)".

Assumptions:

a. If no foxes are present, the rabbits reproduce at a rate proportional to their population, and they are not affected by overcrowding.

b. The foxes eat the rabbits, and the rate at which the rabbits are eaten is proportional to the rate at which the foxes and rabbits interact.

c. Without rabbits to eat, the fox population declines at a rate proportional to itself.

d. The rate at which foxes are born is proportional to the number of rabbits eaten by foxes.

Variables and Parameters:

Indicate which of the following unknowns are *dependent variables, DV, independent variables, IV,* and *parameters, P.*

a. _____ F = population of foxes

b. _____ R = population of rabbits

c. _____ t = time

d. _____ α = growth-rate coefficient of rabbits

e. _____ β = constant of proportionality that measures the fraction of rabbit-fox interactions in which the rabbit is eaten

f. _____ γ = death-rate coefficient of foxes

g. _____ δ = constant of proportionality that measures the benefit to the fox population of rabbit-fox interactions

Note: We assume that $\alpha, \beta, \gamma, \delta$ are all positive.

Formulating the Model:

The rate at which rabbits are eaten is proportional to the rate at which foxes and rabbits interact, so we need a term that models the rate of interaction of the two populations. We want a term that that involves *R* and *F* that increases if either *R* or *F* increases, but it should vanish if either *R=0* or *F=0*. Choose one of the following expressions:

a. _____ $R + F$

b. _____ $R - F$

c. _____ RF

d. _____ $\frac{R}{F}$

e. _____ $\frac{F}{R}$

Express the assumptions mathematically in order to write two differential equations, one describing $\frac{dR}{dt}$, the rate of growth or decline of the rabbit population and the other describing $\frac{dF}{dt}$, the rate of growth or decline of the fox population.

$\frac{dR}{dt}$ = _____ $\frac{dF}{dt}$ = _____

1. In the following predator-prey population models, x represents the prey, and y represents the predators.

A. $\frac{dx}{dt} = 5x - 3xy$, $\frac{dy}{dt} = -2y + \frac{1}{2}xy$

B. $\frac{dx}{dt} = x - 8xy$, $\frac{dy}{dt} = -2y + 6xy$

a. In which system does the prey reproduce more quickly when there are no predators (and equal numbers of prey)? Justify your answer.

b. In which system are the predators more successful at catching prey? Justify your answer.

c. Which system requires more prey for the predators to achieve a given growth rate(assuming identical numbers of predators in both cases)? Justify your answer.