MT 3700 Differential Equations

Linear Equations

Name ____________________

A first order differential equation is linear if it can be written in the form $\frac{dy}{dt} = a(t)y + b(t)$, where $a(t)$ and $b(t)$ are arbitrary functions of t.

Ex.

There are two types of linear differential equations:

1. Homogeneous: $\frac{dy}{dt} = a(t)y$
2. Non-homogeneous $\frac{dy}{dt} = a(t)y + b(t)$

Within non-homogenous there is a special case that we want to distinguish, the case where $a(t) = k$ for some constant k. These differential equations are called constant coefficient equations, $\frac{dy}{dt} = ky + b(t)$.

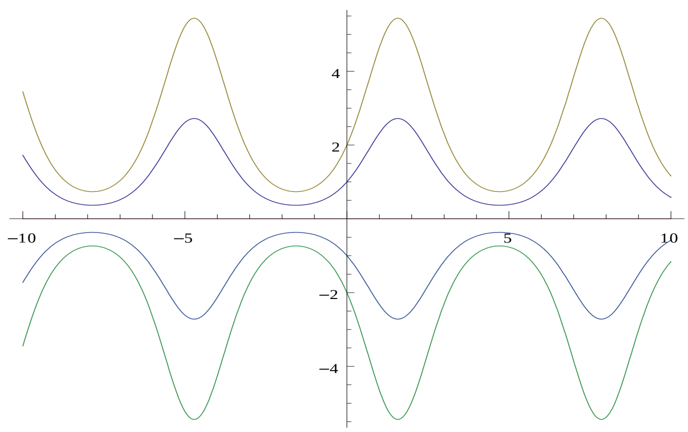
Qn. What is the relationship between linear differential equations and separable differential equations?

- Linearity Principle (homogenous case)

  If $y_h(t)$ is a solution to the homogeneous linear equation $\frac{dy}{dt} = a(t)y$, then any constant multiple of $y_h(t)$ is also a solution. That is, $ky_h(t)$ is a solution for any constant k.
  Notice that $y(t) = 0$ is always a solution.

Argument:

Example: $\frac{dy}{dt} = (\cos t)y.$

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Plot[{e^Sin[t], 0, 2 e^Sin[t], -2 e^Sin[t], -e^Sin[t]}, {t
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- Linearity principle (Non-homogeneous case)

Consider the non-homogeneous equation $\frac{dy}{dt} = a(t)y + b(t)$ and its associated homogeneous equation $\frac{dy}{dt} = a(t)y$

1. If $y_h(t)$ is any solution to the homogeneous equation and $y_p(t)$ is any solution to the non-homogeneous equation, then $y_h(t) + y_p(t)$ is also a solution of the non-homogeneous equation.

2. Suppose $y_p(t)$ and $y_q(t)$ are two solutions to the non-homogeneous equation. Then $y_p(t) - y_q(t)$ is a solution of the associated homogeneous equation.

Therefore, if $y_h(t)$ is nonzero, $ky_h(t) + y_p(t)$ is the general solution of the non-homogeneous equation.

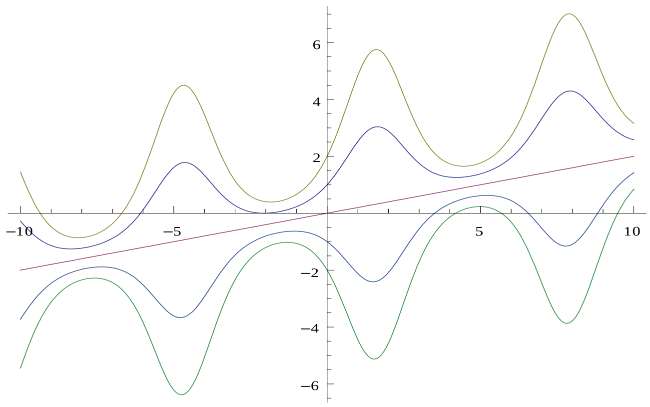
Example: $\frac{dy}{dt} = (\cos t)y + \frac{1}{5}(1 - t\cos t)$.

- Associated homogeneous equation:

- Verify that $y_p(t) = \frac{t}{5}$ is a solution of the non-homogeneous equation.

- The general solution to the non-homogeneous equation is:

- Verify your claim

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Plot[{e^Sin[t] + t/5, t/5, 2 e^Sin[t] + t/5, -2 e^Sin[t] + t/5, - e^Sin
 {t, -10, 10}]
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Solutions to linear differential equations: Three steps for solving linear differential equations

1. Find general solution to the homogeneous equation.
2. Find one particular solution to the non-homogeneous equation.
3. Obtain a general solution of the non-homogeneous equation by adding the general solution of the homogeneous equation to the particular solution of the nonhomogeneous equation.

ISSUE: 2nd step…
SOLUTION: Educated guess...

a) Find a general solution to the differential equation $\frac{dy}{dt} = -4y + 3e^{-t}$

1. Associated homogeneous equation:

$y_h(t) =$ ____________________

2. Guess particular solution to non-homogeneous equation

$y_p(t) =$ ____________________

3. General solution to the non-homogeneous differential equation is:

$y(t) = y_h(t) + y_p(t) =$ ____

b) Find a general solution to the differential equation $\frac{dy}{dt} = -2y + e^{t}$

1. Associated homogeneous equation:

$y_h(t) =$ ____________________

2. Guess particular solution to non-homogeneous equation

$y_p(t) =$ ____________________

3. General solution to the non-homogeneous differential equation is:

$y(t) = y_h(t) + y_p(t) =$ ____

c) Find a general solution to the differential equation $\frac{dy}{dt} = \cos 3t - 2y$

1. Associated homogeneous equation:

$y_h(t) =$ ___________ ________

2. Guess particular solution to non-homogeneous equation

$y_p(t) = \alpha \cos 3t + \beta \sin 3t$

$y_p(t) =$ ___________ ___________ _

3. General solution to the non-homogeneous differential equation is:

$y(t) = y_h(t) + y_p(t) =$ ___________ ___________ ___________ _____