MT 3700 DIFFERENTIAL EQUATIONS
EXISTENCE AND UNIQUENESS OF SOLUTIONS

(1) Consider the following Initial Value Problem:

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0$$

Existence Theorem:

Suppose *f(t,y)* is a continuous function on a rectangle of the form $\{(t, y) | a < t < b, c < y < d\}$ in the *ty*-plane. If (t_o, y_o) is a point in this rectangle, then there exists an $\varepsilon > 0$ and a function *y(t)* defined for $t_0 - \varepsilon < t < t_0 + \varepsilon$ that solves the I.V.P.

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0$$

(a) Summary

- CONDITIONS:

- RESULTS:

(b) Examples: Use the Existence Theorem to determine if the following Initial Value Problems ***(I.V.P.s)*** *have solutions.*

(i) $\frac{dy}{dt} = \frac{1}{y}, \quad y(4) = 0$

(ii) $\frac{dy}{dt} = \frac{1}{y}, \quad y(2) = 3$

(iii) $\frac{dy}{dt} = 1 + y^2, \quad y(0) = 0$

(iv) $\dfrac{dy}{dt} = \dfrac{1}{(y+1)(t-2)}, \quad y(0) = 0$

(v) $\dfrac{dy}{dt} = \dfrac{1}{(y+1)(t-2)}, \quad y(0) = -1$

(2) Consider the following Initial Value Problem:

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0$$

<u>Uniqueness Theorem:</u>

Suppose *f(t,y)* and $\frac{\partial f}{\partial y}$ are continuous functions on a rectangle of the form $\{(t, y) | a < t < b, c < y < d\}$ in the ty-plane. If (t_0, y_0) is a point in this rectangle and if $y_1(t)$ and $y_2(t)$ are two functions that solve the IVP

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0$$

for all *t* in the interval $t_0 - \varepsilon < t < t_0 + \varepsilon$ (where ε is some positive number), then

$$y_1(t) = y_2(t)$$

for all $t_0 - \varepsilon < t < t_0 + \varepsilon$ That is, the solution to the IVP is <u>unique</u>.

(a) *Summary*:

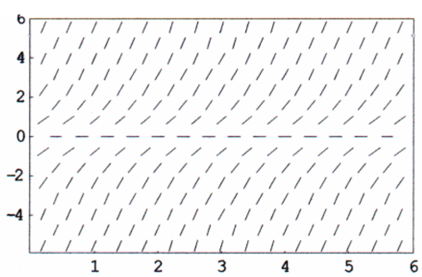
- **CONDITIONS**:

- **RESULTS**:

(b) *Examples*: Use the Existence and Uniqueness Theorems to determine if the following Initial Value Problems **(I.V.P.s)** have solutions and if the solutions are unique.

(i) $\frac{dy}{dt} = 3y^{2/3}, \quad y(2) = 0$

(ii) Demonstrate that there is more than one solution to the IVP in part (i) by sketching the graph of two solutions to the IVP on the slope field given below.



(iii) $\frac{dy}{dt} = 3y^{2/3}, \quad y(2) = 3$

(iv) Go back to the slope field in part (ii) and sketch the solution to this new IVP.