

MT 1800 Calculus I

Worksheet 3.3b – The Product and quotient rules

Purpose: Applications of product and quotient rules

1. The Musée du Louvre has decided to sell one of its paintings and to invest the proceeds.



If the picture is sold between the years 2010 and 2030 and the money, in Euros, from the sale is invested in a bank account earning 5% interest per year compounded annually, then $B(t)$, the balance in the year 2030 depends on the year t in which the painting is sold and the sale price $P(t)$. If t is measured from the year 2010 so that $0 < t < 20$ then the following equation applies.

$$B(t) = P(t)(1.05)^{20-t}$$

- (a) Explain in detail why $B(t)$ is given by this formula.

(b) Show that the formula for $B(t)$

$$B(t) = P(t)(1.05)^{20-t}$$

is equivalent to the following formula.

$$B(t) = (1.05)^{20} \left(\frac{P(t)}{(1.05)^t} \right)$$

(c) What are the units of $P'(t)$? What is the practical meaning of this derivative?

(d) What are the units of $B'(t)$? What is the practical meaning of this derivative?

(e) Conjectures:

What can you speculate about the sign of $P'(t)$? Explain in detail the reasoning behind your speculations.

What can you speculate about the sign of $B'(t)$? Explain in detail the reasoning behind your speculations.

(e) Find $B'(t)$ if

$$B(t) = (1.05)^{20} \left(\frac{P(t)}{(1.05)^t} \right)$$

(f) Find $B'(10)$, given that $P(10) = 150000$ and $P'(10) = 5000$. Interpret the meaning of this derivative.

(g) When would the best time to sell be? Why? (You can be creative, but you need to support your arguments with conclusions you have drawn above.)

2. The males in the frog species *Eleutherodactylus coqui* (found in Puerto Rico) take care of their brood.



While they protect the eggs they cannot find other mates, and therefore cannot increase their number of offspring. On the other hand, if they do not spend enough time with their brood, then the offspring might not survive. Simple mathematical models are used to give the proportion of offspring per unit of time, $w(t)$, as a function of the probability of hatching if time t is spent brooding, $f(t)$, and the cost C associated with the time spent searching for other mates. Given the following relationship

$$w(t) = \frac{f(t)}{C + t}$$

- a. Find the derivative $w'(t)$.
- b. What are the units of $w'(t)$? Interpret it in biological terms.