

MT 1800 CALCULUS I
Worksheet 3.10 – Theorems about differentiable functions

Name _____

Instructions: Work in groups. Each member of the group should turn in his/her own worksheet at the end of class.

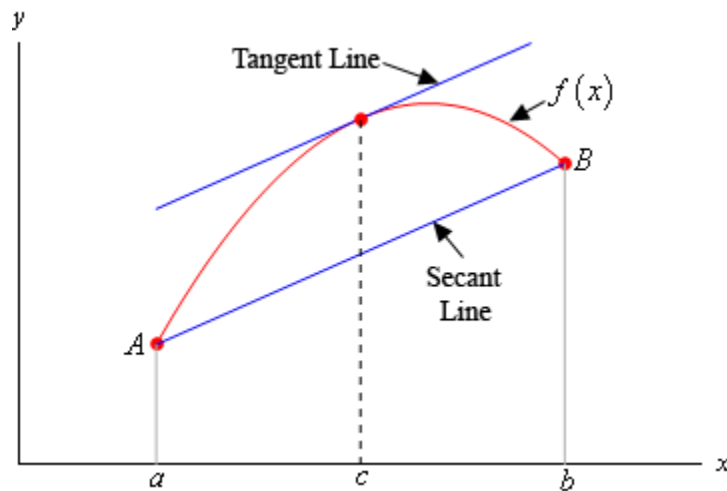
Mean Value Theorem:

If f is continuous on $a \leq x \leq b$ and differentiable on $a < x < b$, then there exists a number c in (a, b) , such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

That is, there exists a number c in (a, b) such that the instantaneous rate of change at c is equal to the average rate of change of the function on the interval (a, b) .

Or, equivalently, there exists a number c in (a, b) such that the slope of the tangent line at c equals the slope of the secant through $(a, f(a))$ and $(b, f(b))$.



1. Consider $f(x) = e^x$ on $[1,4]$.

$f(x)$ is continuous on $(1,4)$ ✓

$f(x)$ is differentiable on $[1,4]$ ✓

Mean Value Theorem applies. ☺

Therefore, we can guarantee that there exist a $c \in [1,4]$ such that:

$$f'(c) = \frac{f(4) - f(1)}{4 - 1} = \frac{e^4 - e}{3} = 17.29$$

Find c .

2. Consider $f(x) = x^3 - 9x^2 - 48x + 52$ on $[-4.4418, 0.9361]$.

Verify that the Mean Value Theorem applies.

Find $c \in [-4.4418, 0.9361]$ guaranteed by the theorem.

3. Does the Mean Value Theorem apply to the following functions on the given interval?

a. $f(x) = \frac{1}{x}$ on $[-2,2]$

Yes/No If no, Why?

b. $f(x) = \frac{1}{x}$ on $[2,4]$

Yes/No If no, Why?

c. $f(x) = |x|$ on $[2,4]$

Yes/No If no, Why?

d. $f(x) = |x|$ on $[-1,3]$

Yes/No If no, Why?

The case of the speeding mathematician

The Centro de Investigación en Matemáticas (CIMAT) is located in the city of Guanajuato. Suppose a mathematician from the National Autonomous University of Mexico (UNAM), which is located in Mexico City, has been invited to give a talk at the CIMAT at 3:00 pm on November 3rd.

Because he's running a bit late, he decides along the way to take the toll highway for the remaining 200 km. He gets his highway ticket stamped at exactly 1:00 pm.

The maximum speed along the toll highway is 120 km/hr. Tickets are given to people on a regular basis but the official giving a ticket has to provide evidence of the speed violation.

Our mathematician arrives in Guanajuato at exactly 2:30. There he sees one of his graduate students working at the toll booth. He hands her his toll ticket and, with a grin, she returns it with an extra \$25.00 traffic fine!!!

The professor is a bit shocked at first and then realizes what is going on: "Mean Value Theorem ☺!" The graduate student grins even more...

Explain how this mathematics graduate student can prove her case in court. Write out the details carefully.