

MT 1800 Calculus I
Worksheet 2.3: The Derivative Function—Power Functions

- **Purpose:** To examine the instantaneous rates of change of $f(x) = x^2$ at various points, and to develop a general formula for derivatives of power functions.

The instantaneous rate of change of a function f at a point $x = a$, also known as the derivative of f at $x = a$, is denoted by $f'(a)$. Recall the definition:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

In a previous worksheet, we saw that we can always estimate $f'(a)$ graphically, and compute it numerically or algebraically.

Now we will use an applet (from the collection of Math Applets for Calculus at SLU) to generate a conjecture on a general formula for the derivative of a power function at a point.

Go to the page: <http://www.slu.edu/classes/maymk/SecantTangent/SecantTangent.html>

This applet is designed for the visual exploration of secant and tangent lines. In particular it lets the user compare the graph of a function with either the line secant at x values X_0 and $X_0 + dX$, or with the left secant at values $X_0 - dX$ and X_0 , or with the line secant at X_0 with slope obtained from the balanced difference quotient from the line secant at $X_0 - dX$ and $X_0 + dX$.

Let's estimate a few derivatives in order to draw a conjecture.

1. Let's start with $f(x) = x^2$.

- a. Empty the first cell in the right column and enter : $f(x) = x^2$

We want to estimate the instantaneous rate of change of f at different values for x and see if we can establish a pattern.

- b. Use the slopes of secants for very small values of dX to approximate the slope of the tangent and estimate the following derivatives.

$$f'(4) \cong \underline{\hspace{2cm}}$$

$$f'(2) \cong \underline{\hspace{2cm}}$$

$$f'(1) \cong \underline{\hspace{2cm}}$$

$$f'(0) \cong \underline{\hspace{2cm}}$$

$$f'(-1) \cong \underline{\hspace{2cm}}$$

$$f'(-2) \cong \underline{\hspace{2cm}}$$

$$f'(-4) \cong \underline{\hspace{2cm}}$$

- c. Guess : $f'(8) \cong \underline{\hspace{2cm}}$

- d. Guess : $f'(-6) \cong \underline{\hspace{2cm}}$

- e. Conjecture: For $f(x) = x^2$, $f'(x_0) \cong \underline{\hspace{2cm}}$

2. Prove your conjecture by using the limit definition of the derivative.

$$\text{Recall } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

3. Now let's change things a bit and consider $f(x) = 4x^2$.

- Empty the first cell in the right column and enter : $f(x) = 4 * x^2$
- Use the slopes of secants for very small values of dX to approximate the slope of the tangent and estimate the following derivatives.

$$f'(4) \cong \underline{\hspace{2cm}}$$

$$f'(2) \cong \underline{\hspace{2cm}}$$

$$f'(0) \cong \underline{\hspace{2cm}}$$

$$f'(-1) \cong \underline{\hspace{2cm}}$$

$$f'(-2) \cong \underline{\hspace{2cm}}$$

$$f'(-4) \cong \underline{\hspace{2cm}}$$

c. Guess : $f'(8) \cong \underline{\hspace{2cm}}$

d. Guess : $f'(-6) \cong \underline{\hspace{2cm}}$

e. Conjecture: For $f(x) = 4x^2$, $f'(x_0) \cong \underline{\hspace{2cm}}$

- f. Do as many more calculations as you need in order to formulate the following

Conjecture:

When $f(x) = cx^2$, where c is any real number, $f'(x_0) \cong \underline{\hspace{2cm}}$

4. Now let's consider $f(x) = x^3$.

- Empty the first cell in the right column and enter : $f(x) = x^3$
- Use the slopes of secants for very small values of dX to approximate the slope of the tangent and estimate the following derivatives.

$$f'(4) \cong \underline{\hspace{2cm}}$$

$$f'(2) \cong \underline{\hspace{2cm}}$$

$$f'(0) \cong \underline{\hspace{2cm}}$$

$$f'(-2) \cong \underline{\hspace{2cm}}$$

$$f'(-4) \cong \underline{\hspace{2cm}}$$

- Guess : $f'(8) \cong \underline{\hspace{2cm}}$

- Guess : $f'(-6) \cong \underline{\hspace{2cm}}$

- Conjecture: For $f(x) = x^3$, $f'(x_0) \cong \underline{\hspace{2cm}}$

4. Try now $f(x) = 4x^3$.

- Empty the first cell in the right column and enter : $f(x) = 4 * x^3$
- Use the slopes of secants for very small values of dX to approximate the slope of the tangent and estimate the following derivatives.

$$f'(4) \cong \underline{\hspace{2cm}}$$

$$f'(2) \cong \underline{\hspace{2cm}}$$

$$f'(0) \cong \underline{\hspace{2cm}}$$

$$f'(-1) \cong \underline{\hspace{2cm}}$$

$$f'(-4) \cong \underline{\hspace{2cm}}$$

- Guess : $f'(8) \cong \underline{\hspace{2cm}}$

- Guess : $f'(-6) \cong \underline{\hspace{2cm}}$

- e. Conjecture: For $f(x) = 4x^3$, $f'(x_0) \cong$ _____
- f. Do as many more calculations as you need in order to formulate the following

Conjecture:

When $f(x) = cx^3$, where c is any real number, $f'(x_0) \cong$ _____

5. Look at your conjectures in 1f and 3f.

Make another conjecture:

When $f(x) = x^n$, where n is any positive integer,

$f'(x_0) \cong$ _____

Your conjecture above is called the Power Rule and it actually holds for any real value of n .

6. Applying the power rule.

Let $f(x) = x^4$. Find the equation of the line tangent to the graph at $x = 2$.

Tangent line at $x = 2$: _____