MT 1800 Calculus I
Worksheet 1.3b/1.4 – *Inverse Functions and logarithmic functions*

**Name: _____**

Purpose:

- To learn about characteristics of inverse functions.
- Define logarithmic functions.
- To solve equations involving exponentials and logarithms.
- To investigate applications of logarithms.

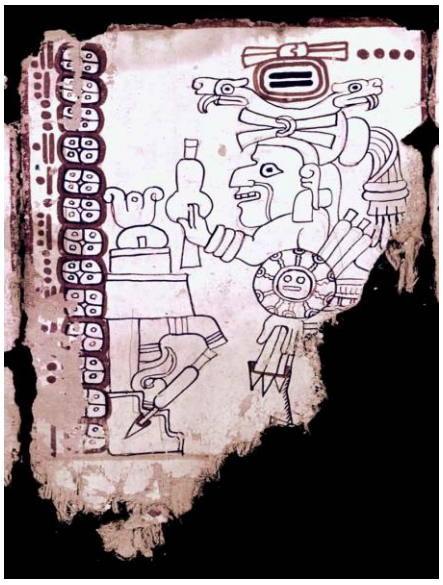
Procedure: We will work on this worksheet together as a class.

## Problem: Real or Fake?

**You have been hired to decide if the following Mayan Codex is authentic or a fake.**

*Grolier Codex*

*While other three (Maya) codices were known to scholars since the 19th century, the Grolier Codex only surfaced in the 1970s. This fourth Maya codex was said to have been found in a cave, but the question of its authenticity has still not been resolved to everybody's satisfaction. The codex is really a fragment of 11 pages. It is currently in a museum in Mexico, but is not on display to the public. Scanned photos of it are available on the web. The pages are much less detailed than any of the other codices. Each page shows a hero or god, facing to the left. At the top of each page is a number. Down the left of each page are what appear to be a list of dates. (Excerpt from Wikipedia)*



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## Methodology:

### *Radiocarbon dating*

*From: http://id-archserve.ucsb.edu/anth3/courseware/Chronology/08_Radiocarbon_Dating.html*

*Radiocarbon, or Carbon-14, dating is probably one of the most widely used and best known absolute dating methods. It was developed by J. R. Arnold and W. F. Libby in 1949, and has become an indispensable part of the archaeologist's tool kit since. Its development revolutionized archaeology by providing a means of dating deposits independent of artifacts and local stratigraphic sequences. This allowed for the establishment of world-wide chronologies.*

### *C-14 Decay Profile*

*The C-14 within an organism is continually decaying into stable carbon isotopes, but since the organism is absorbing more C-14 during its life, the ratio of C-14 to C-12 remains about the same as the ratio in the atmosphere. When the organism dies, the ratio of C-14 within its carcass begins to gradually decrease. The rate of decrease is 1/2 the quantity at death every 5,730 years. That is the half-life of C-14.*

*This dating method is accurate to +-130 years.*

A. Can you find an exponential function that describes the decay of C-14 and that we could use to find out how old the Codex really is?

   Let the amount of $C^{14}$ at time $t$ be denoted by $W(t)$, with $W_0 = W(0)$.

$W(t) =$ (Eq. 1)

Can we solve for t?

Issue:

## Overcoming the issue / Developing some tools

**One to one functions**
Definition
A function is said to be a one-to-one function if it has the property that whenever $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$.

**Ex.**

**Inverse Functions**

Definition
Let $f: A \to B$ be a one-to-one function with range $f(A)$. The inverse function $f^{-1}$ has domain $f(A)$ and range $A$ and is defined by

$$f^{-1}(y) = x \text{ if and only if } y = f(x), \text{ for all } y \in f(A).$$

**Logarithmic functions**

Definition
The inverse of $f(x) = a^x$ is called the logarithm of base $a$ and is written $f^{-1}(x) = log_a x$.

Pictures:

Domain:

Range:

We will concentrate on the logarithms of base 10 and e.

Common log: $logx := log_{10} x$ $(c = log_{10} x$ iff $10^c = x)$

Natural log: $lnx := log_e x$ $(c = log_e x$ iff $e^c = x)$

Practice:

**Equations involving exponentials**
For properties of logs (See page 25 in your book!)

1. $3^x = 11$

2. $Q = Q_0 a^{nt}$

3. $P = 174(0.9)^t$

## Rewriting the model in terms of continuous growth rate

In page 2, we solved the equation:

$$\frac{1}{2} = a^{5730}$$

and obtained that $a = 0.99987$

B. Rewrite $a$ in terms of the natural base $e$.

$a=$ _____

C. If the amount of $C^{14}$ at time $t$ is denoted by $W(t)$, with $W_0 = W(0)$, then written in terms of the natural base $e$ , Eq. 1 becomes:

$W(t) =$ _____

Notice that our model is now in the form:

$$W(t) = W_0 e^{-\lambda t}, t \geq 0.$$

Where $\lambda > 0$ denotes the decay rate, in this case, expressed in terms of the half-life of the material, which is the length of time that it takes for half of the material to decay.

## Back to our initial problem of determining if the Grolier Codex is a fake.

D. A sample of the Grolier Codex was sent Teledyne Isotopes to measure the current amount of C-14 in the sample. The lab reported that 86% of the original amount of C-14 remains in the sample. How old is the Codex?

_____

E. Is it an original? _____