MT 1800 Calculus I

Worksheet 1.2b – *Exponential Functions: Another Defining Characteristic*

Name: ______________________ **Due date: Tuesday, September 2nd, 08.**

Purpose:

- To learn about mathematical models that describe biological growth.
- To characterize the type of function whose rates of change are proportional to the function.

Procedure: The work here serves as your notes on this topic. (Note that the ideas explored in this worksheet are NOT covered in your text, but you will be responsible for applying these ideas in future coursework.)

Problem: Biological Growth

We consider the growth of fruit flies in a favorable laboratory environment: unlimited food, unlimited space, and no predators. Our objective is to characterize the growth rate of fruit flies and to find an *approximating* function (a model which fits the data fairly well) that we can describe in some simple way (e.g., by a formula) and that we can use to estimate the population of fruit flies at any (reasonable) time, without actually counting the flies.

The real population function assumes only integer values. (We do not count pieces of flies!) However, we allow our approximating function to assume fractional values. When we interpret our estimates from our approximating function, we will have to remember not to be too impressed by a prediction of, say, 788.025 flies on day 15.

Here are the data obtained by counting flies on various days among a 16 day period.

Day, (t)	***Flies, p(t)***	***Daily Growth Rate,*** $\Delta p / \Delta t$	
0	111	----------------	-------------
1	122		
3	147		
4	161		
7	214		
9	258		
13	394		
16	534		

1. Calculate the daily growth rate of this population of fruit flies for each of the seven days listed and record your answers in the column labeled *Daily Growth Rate*.

2. What can you say about the daily rate of growth of the population? In particular, is it constant, increasing, decreasing?

3. Biologists argue that, for populations of this type, the rate of growth should be proportional to the population. How can we test whether the data in hand supports this theory?

4. Use the blank fourth column provided in your data table to carry out your test.

5. Assuming there is one, what is your best estimate of the proportionality constant?

6. We need to know what sort of function has a rate of change proportional to the function, itself. Decide which of the following functions has a rate of change proportional to the function by conducting your test on the function at times $t = 0,1,2,3$. Show your calculations (by completing the tables provided) and conclusions for each of the six functions below.

a. $f(t) = t^3$

t	$f(t)$	Rate of change $\frac{\Delta f}{\Delta t}$	$\frac{\Delta f / \Delta t}{f}$
0			
1			
2			
3			

Is the rate of change of f(t) proportional to f(t) itself? (YES or NO) _______

b. $f(t) = 3^t$

t	$f(t)$	Rate of change $\frac{\Delta f}{\Delta t}$	$\frac{\Delta f / \Delta t}{f}$
0			
1			
2			
3			

Is the rate of change of f(t) proportional to f(t) itself? (YES or NO) _______

c. $f(t) = 4(3^t)$

t	$f(t)$	Rate of change $\frac{\Delta f}{\Delta t}$	$\frac{\Delta f / \Delta t}{f}$
0			
1			
2			
3			

Is the rate of change of f(t) proportional to f(t) itself? (YES or NO) _______

d. $f(t) = 3t + 5$

t	$f(t)$	Rate of change $\frac{\Delta f}{\Delta t}$	$\frac{\Delta f / \Delta t}{f}$
0			
1			
2			
3			

Is the rate of change of f(t) proportional to f(t) itself? (YES or NO) _______

e. $f(t) = t^2$

t	$f(t)$	Rate of change $\frac{\Delta f}{\Delta t}$	$\frac{\Delta f / \Delta t}{f}$
0			
1			
2			
3			

Is the rate of change of f(t) proportional to f(t) itself? (YES or NO) _______

f. $f(t) = 5t^2$

t	$f(t)$	Rate of change $\frac{\Delta f}{\Delta t}$	$\frac{\Delta f / \Delta t}{f}$
0			
1			
2			
3			

Is the rate of change of f(t) proportional to f(t) itself? (YES or NO) _______

g. $f(t) = e^{3t}$

t	$f(t)$	Rate of change $\frac{\Delta f}{\Delta t}$	$\frac{\Delta f / \Delta t}{f}$
0			
1			
2			
3			

Is the rate of change of f(t) proportional to f(t) itself? (YES or NO) _______

7. What class of functions has the property that the rate of change of the function is proportional to the function, itself?

8. Find an appropriate modeling function for the biological growth data given on the first page.

Model: $P(t) =$

9. Test the model. Use your model to calculate the population of fruit flies

   after 9 days: ___________________

   after 16 days: __________________

   How well does the model fit the data?

10. Use the model. Predict the population of fruit flies

    after 20 days: __________________

    after 23 days: __________________