

MT 1800
Calculus 1 - Test 2

Name _____

1. (12 points) Match the points labeled on the curve in Figure 1 with the given slopes.

Slope	Point
-3	
-1	
0	
1/2	
1	
2	

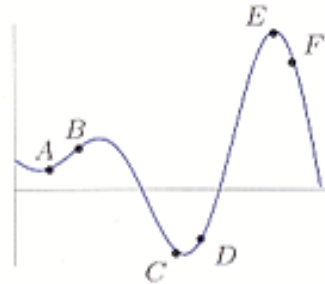


Figure 1

2. (10 points) Find the derivative of the following function by using the definition. Pay careful attention to notation while you do this.

$$f(x) = 6x^2 + 5 \text{ at } x = 2$$

3. (12 points) Find a formula for the derivative of the following functions using the power rule.

a. $f(x) = 14x - 8$ $f'(x) =$ _____

b. $f(x) = x^6$ $f'(x) =$ _____

c. $f(x) = x^{2.8}$ $f'(x) =$ _____

d. $f(x) = \frac{1}{x^3}$ $f'(x) =$ _____

e. $f(x) = \sqrt{x}$ $f'(x) =$ _____

4. For the function: $f(x) = 3x^2$

a. (2 points) Find the average rate of change of $f(x)$ from $x = 10$ to $x = 12$.

b. (2 points) Find a general formula for the instantaneous rate of change of $f(x)$ at a generic point $x = x_0$.

c. (1 point) Find the instantaneous rate of change of $f(x)$ at $x = 11$.

d. (3 points) Find the equation of the tangent line to the graph of the function at $x = 11$.

5. (4 points) Use Figure 2 to fill in the blanks in the following statements about the function g at point B .

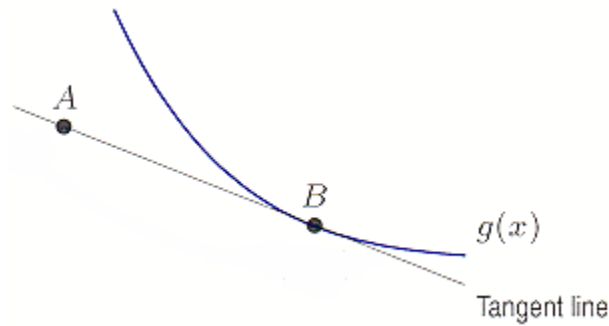


Figure 2

where $A = (1, 12)$ and $B = (2, 10)$

- a. $g(2) =$ _____
- b. $g'(2) =$ _____
6. (8 points) The temperature, T , in degrees Fahrenheit, of a room temperature yam placed in a functioning refrigerator is represented by $T = f(t)$, where t is the time in minutes since the yam was put in the refrigerator.
- (a) What is the sign of $f'(t)$?
- (b) Why?
- (c) What are the units of $f'(20)$?
- (d) What is the practical meaning of the statement $f'(20) = -2$?

7. (12 points) Figure 3 is the graph of f' , the derivative of a function f .

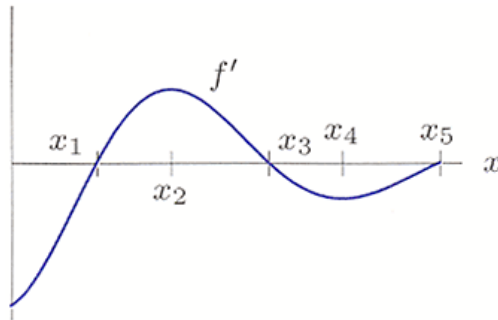


Figure 3 Graph of f' , not f

(a) On what interval(s) is the function f increasing?

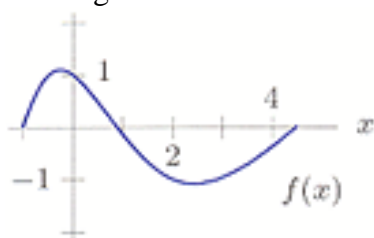
(b) On what interval(s) is the function f decreasing?

(c) On what interval(s) is the function f concave up?

(d) On what interval(s) is the function f concave down?

8. (6 points) For the function graphed in Figure 4, are the following nonzero quantities positive or negative?

Figure 4



(a) $f(2)$

(b) $f'(2)$

(c) $f''(2)$

9. (6 points) The distance of a car from its initial position t minutes after setting out is given by $s(t) = 5t^2 + 4$ kilometers.

(a) What is the car's velocity at time $t=3$?

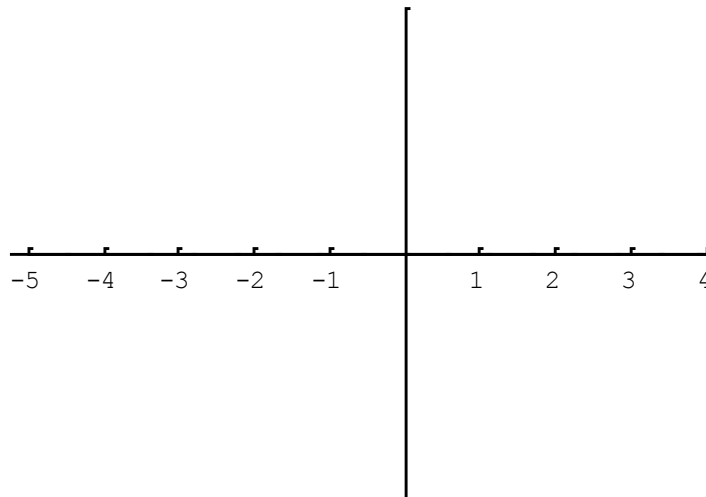
(b) Velocity Units:

(c) What is the car's acceleration at time $t=3$?

(d) Acceleration Units:

10. (6 points) Draw a function f that satisfies the following conditions.

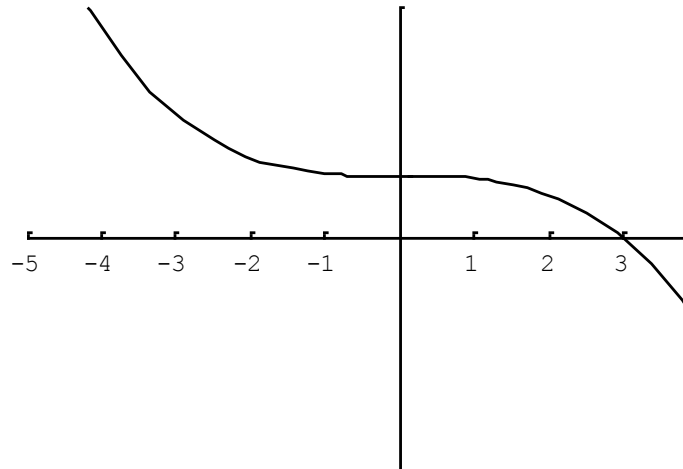
$$\begin{aligned}f(3) &= 27 \\f'(3) &= 3 \\f''(x) &< 0 \text{ for } x \geq 3\end{aligned}$$



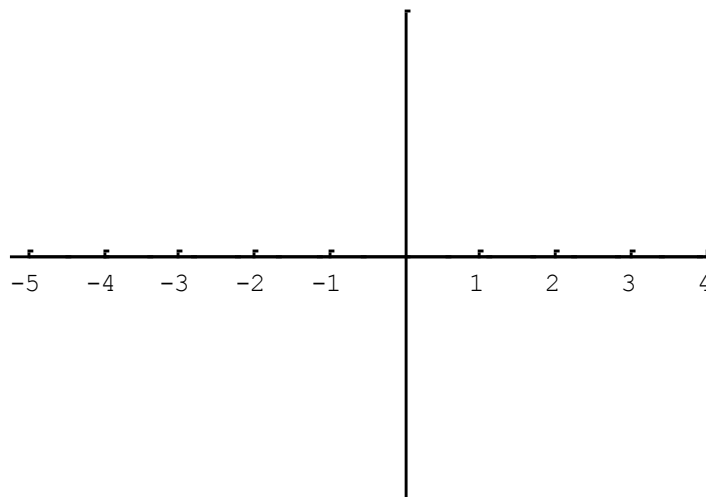
11. (6 points) Below, in Figure 5, you are given the graph of a function, $g(x)$. Sketch a possible graph of its derivative function, $g'(x)$.

Figure 5

$$y = g(x)$$

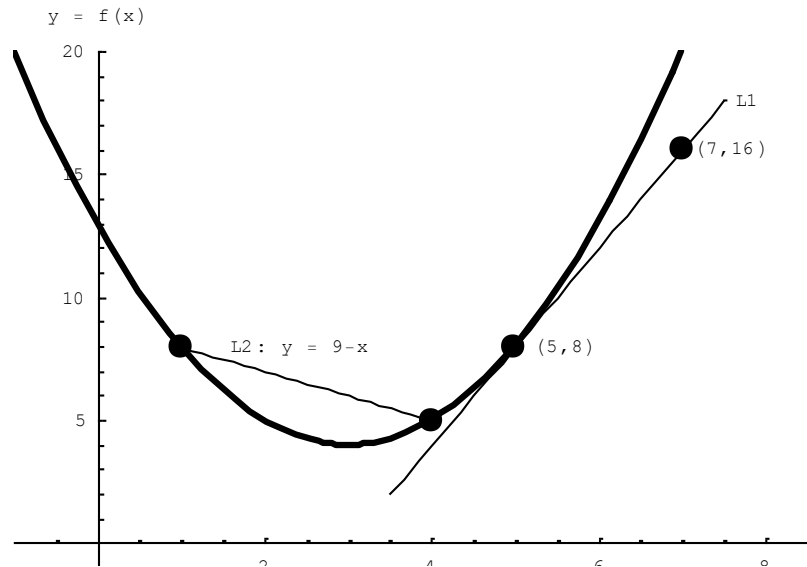


$$y = g'(x)$$



12. (10 points) Consider the graph of the function $f(x)$ given below in Figure 6. Note that L2 is the line that connects $(1, f(1))$ with $(4, f(4))$, and L1 is the tangent line to $f(x)$ at $x = 5$.

Figure 6



Carefully study the graph and complete the following statements.
Provide numerical answers to parts (a) – (d) below.

- The **average rate of change** of $f(x)$ from $x = 1$ to $x = 4$ is _____.
- The **instantaneous rate of change** of $f(x)$ at $x = 5$ is _____.
- $\frac{f(4) - f(1)}{3} =$ _____.
- $\lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h} =$ _____.
- L2 is called the _____ line of $f(x)$ from $x = 1$ to $x = 4$.

Extra credit for correct mathematical language and notation:

Mathematics is a language, and use of proper wording and notation is essential to clear communication. This checklist gives you an opportunity to review your work and make sure you have used correct mathematical language and notation.

5 checks: +2

4 checks: +1

- ___ 1. Equal sign used properly. The expressions on both sides of this symbol must indeed be equal to each other.

- ___ 2. Correct use of variable. Be sure that you are consistent in your use of variables. State your answers in terms of the variables given in the problem. For example, if the problem asks you to find a model $P(t)$, for some population, your model must use the variables P and t .

- ___ 3. Correct limit notation. $\lim_{x \rightarrow a} f(x) = L$. Limits must have an argument, a subscript, and an answer.

- NA4. Correct notation for antiderivatives (integrals): $\int f(x)dx = \text{answer}$. Correct notation begins with the antiderivative symbol, has an expression in the middle, and ends with dx , dt , etc. (according to the variable—see item 2 above). The answer does not include the symbol.

- NA5. Answers to antiderivatives end with $+C$. This is essential for solving IVP's correctly.

- ___ 6. Correct use of fractions. It must be clear what expression is on the top and what expression is on the bottom. Use horizontal fraction bar or parentheses were appropriate.

- ___ 7. Written descriptions should be given in complete sentences.

- ___ 8. Your work must be neat. Neatness always counts.

- ___ 9. Use parentheses where necessary. For example, $4t - 5$ and $4(t - 5)$ are not equal.