

## Chapter 4 - Applications of the derivative 3

### Optimization and modeling

The idea in this section, as usual, is to figure out how we can use methods that we already know, to solve more general problems.

In many of the problems in this section you will be maximizing or minimizing a function of more than one variable. With the methods we have developed so far you can find extrema for functions of one variable.

What to do? Ideas?

A generic problem could look like this:

Max  $f(x,y)$

Subject to  $g(x,y)=a$

### Guidelines for optimization problems

1. Read the question carefully.
2. Think about the question.

#### **What are you supposed to do? What is the information they give you?**

3. Draw a picture and label quantities.
4. Now formalize what you found out in 1. Write down any expression(s) you have which relate the quantities. USE THE LABELS YOU CHOSE!
5. Write down an expression for the quantity you want to optimize. Generally, that expression will depend on more than one variable. We will call this your objective function.

#### **Hmm... How does one optimize functions of more than one variable? Could we possibly convert the function into a function of only one variable by using the other relations... lets try!**

6. Use the relations from step 3 to eliminate a variable in your objective function.

#### **You have converted a problem you didn't know how to solve into one you do know how to solve!!!**

7. Differentiate the quantity to be optimized with respect to the remaining variable. Set that derivative to zero and solve.
8. Extrema occur at critical points (which you just found) or boundary points. Find the value at boundary points.
9. The max/min occurs at the smallest/largest of the critical point(s) and the boundary points.
10. Sanity-check your answer.

**Example: Find the dimensions of the rectangle of maximum area with perimeter 1000 feet.**

1. **Find the dimensions of the rectangle of maximum area with perimeter 1000 feet.**

2. Thoughts:

3. Draw a picture of a rectangle, labeling the length  $\ell$  and width  $w$ .

4.  $P(w,\ell) = \underline{\hspace{2cm}}$

$A(w,\ell) = \underline{\hspace{2cm}}.$

5. The quantity to be maximized is  $\underline{\hspace{2cm}}.$

6. We can rewrite the quantity to be maximized in terms of one variable, lets choose  $w$ .

Then  $\underline{\hspace{1cm}}(\underline{\hspace{1cm}}) = \underline{\hspace{2cm}}.$

7. Now we can use our "old techniques" 😊

Differentiating gives us

$\underline{\hspace{1cm}}(\underline{\hspace{1cm}}) = \underline{\hspace{2cm}}$

8. The smallest  $w$  can be is  $\underline{\hspace{2cm}}.$  This gives a rectangle of area  $\underline{\hspace{2cm}}.$

The largest  $w$  can be (due to the perimeter constraint) is  $\underline{\hspace{2cm}}.$

This gives a rectangle of area  $\underline{\hspace{2cm}}.$

9. So, the maximum area occurs when  $w = \underline{\hspace{2cm}}$

10. Sanity check:

### Problem 1.

You are to make a box with square base and no top. You will (as guided below) find the dimensions that minimize the surface area of the box if the volume of the box is to be  $1000 \text{ cm}^3$ .

Follow the 10 steps listed in the guidelines. Here is a bit of help to get you through this one.

- What do you call the three dimensions (front-to-back distance, left-to-right distance, bottom-to-top distance) of the box? The variable names are your choice.
- What is the volume of a box in terms of those three lengths? Write an equation.
- What does “square base” mean in terms of those three lengths?
- Write an equation for  $V = 1000$ , in terms of those lengths. How many variables does  $V$  depend on?
- What is the surface area  $A$  of the box? Remember we’re not including the top. There are four sides, and the bottom. Those are all rectangles. What is the area of each? Remember that areas add. What is the sum of the 5 areas?
- What is the name of the quantity we are asked to optimize?
- How many variables does that quantity depend on?
- Can those variables take on any value, or are they related somehow? Write down an equation involving them. Then solve that equation for one of the variables.
- Plug that variable back into the area equation. Area should now depend on just one variable.
- Differentiate that equation and set it to zero.
- Solve for the remaining variable.
- Now find the other dimensions of the box.
- Does everything look reasonable? Why or why not?

Name \_\_\_\_\_

Please turn in the following two problems next Monday, Nov 17.

**Problem 2.**

You are to fence a rectangular area. The fencing for the left and right sides costs \$2/foot; the fencing for the front and back sides costs \$3/foot. Find the dimensions of the rectangular area that result in least cost, if the area is to be 3200 square feet.

Show each step.

**Problem 3.**

What are the dimensions of an aluminum can that holds  $40 \text{ in}^3$  of juice and that uses the least material (i.e. aluminum)? Assume that the can is cylindrical and is capped on both ends.

Show each step.