Challenge Problems Chapter 4

1. Water is being poured into a conical reservoir at the rate of pi cubic feet per second. The reservoir has a radius of 6 feet across the top and a height of 12 feet. At what rate is the depth of the water increasing when the depth is 6 feet?

2. A man 6 feet tall is walking toward a lamppost 20 feet high at a rate of 5 feet per second. The light at the top of the lamppost (20 feet above the ground) is casting a shadow of the man. At what rate is the tip of his shadow moving and at what rate is the length of his shadow changing when he is 10 feet from the base of the lamppost?

3. If a triangle has sides of length a, b, and c, then *Hero's Formula* enables you to find the area of the triangle without knowing its height. The formula says to set

$$s = \frac{a + b + c}{2}$$

Then the area, A, of the triangle is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

The problem here is, given that the lengths of sides b and c are fixed and constant, find the length for side a that maximizes the triangle's area. **Hint:** Since area is always positive, you can maximize area by maximizing the *square* of the area.

4. Suppose you are in the business of manufacturing and selling bow-ties. Your marketing research indicates that the number of bow-ties you will sell each month is related to the price you charge, x, according to the function:

$$n(x) = 40000 - 800x$$

This is called the *demand function*. So, for example, if you gave them away, you would only get rid of 40,000 bow-ties per month. At the opposite end, if you charged $50 apiece for them, you would sell none. You are looking for a happy medium in between where you will earn some profit. It turns out it costs you $1000 per month to rent the building where you have your bow-tie factory. It also costs you $2 per bow-tie in material and labor to produce them. If profit is revenue minus cost, determine the price, x, you should charge in order to maximize your profit.