MT 1800 – Calculus I
Introduction to Chapter 4—Applications of the Derivative

Name: _____ Name: _____

Now that we know how to calculate and interpret derivatives, we would like to use them to better understand functions and to solve some applied problems.

- **Purpose**: To practice your abilities to use first and second derivatives to analyze the graph of a function.

- **Procedure**: Turn in this worksheet with your answers completed and a copy of your *Mathematica* notebook attached. (Each lab team turns in one copy).

Suppose f is a differentiable function. Recall that the graph of $y = f(x)$

- crosses the x-axis when $f(x) = 0$
- has a horizontal tangency when $f'(x) = 0$
- changes concavity (inflection point) when $f''(x) = 0$

Example:

$$f(x) = -40 - 18x + 3x^2 + x^3$$

1. Enter *f(x)* as a *Mathematica* function.
   Recall that functions in *Mathemtica* are defined using the command **f [x_]:=**

2. What are the values of x where $f(x)$ crosses the x-axis (roots)?

   The **Solve** command in *Mathematica* can help you find all the **roots** of *f(x)*.
   Type: **Solve[f[x] = = 0,x]**
   Record your answers below:

   $f(x) = 0$ for _____

3. Find $f'(x)$ using *Mathematica*.

   (Remember that you just have to type **f'[x]** and press numerical enter to find the derivative of a function)

   $f'(x) =$ _____

4. What are the values of x at which $f(x)$ has a horizontal tangency?

   Use the **Solve** command to find these values. That is, type **Solve[f'[x] = = 0,x]**

   $f'(x) = 0$ when _____

5. Use Mathematica to find $f''(x)$

   $f''(x) =$_____

6. What are the values of x where $f(x)$ changes concavity?

   Use the **Solve** command again to find these values.

   $f''(x) = 0$ when _____

Furthermore, the graph of $y = f(x)$ is

- above the x-axis when $f(x) > 0$
- below the x-axis when $f(x) < 0$
- increasing when $f'(x) > 0$
- decreasing when $f'(x) < 0$
- concave up when $f''(x) > 0$
- concave down when $f''(x) < 0$

Note:

To find out where a function is positive and were it is negative. Use the zeros to divide the region of interest into sub-regions, choose a point in each sub-region arbitrarily and evaluate the function. The sign of the outcome is the sign of the function in that sub-region.

Returning to our example, write your answers using interval notation:

- f is above the x-axis for
- f is below the x-axis for
- f is increasing for
- f is decreasing for
- f is concave up for
- f is concave down for

We say p is a <u>critical point</u> of f if $f'(p)=0$ or $f'(p)$ is undefined yet $f(p)$ is defined.

Example:

7. Use *Mathematica* to find any critical points for

   a. $g(x)=x^2-x-12$

   b. $h(x)=\sqrt[3]{x+1}$

<u>Local Extrema – First derivative test p.168</u>

We say f has a <u>local maximum</u> at p if

	$f'(x)>0$	p	$f'(x)<0$
$f(x)$	↗	p	↘

We say f has a <u>local minimum</u> at p if

	$f'(x)<0$	p	$f'(x)>0$
$f(x)$	↘	p	↗

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8. Identify if any of the critical numbers is an extremum in the examples above.

Record your findings:

   a. About $g(x)=x^2-x-12$

	$f'(x)$	p	$f'(x)$
$f(x)$		p	

   b. About $h(x)=\sqrt[3]{x+1}$

	$f'(x)$	p	$f'(x)$
$f(x)$		p	

Global extrema

We say f has a global maximum at p if $f(p) \geq f(x)$ for all x.
The global max is the largest value of the function in an interval.

We say f has a global minimum at p if $f(p) \leq f(x)$ for all x.
The global min is the smallest value of the function in an interval.

Example:

9. Find global extrema for $g(x) = x^2 - x - 12$ on the interval $[-5,5]$.

10. Find global extrema for $h(x) = \sqrt[3]{x+1}$ on the interval $[-4,3]$.

11. **Use calculus** to analyze the function $f(x) = x^3 - 9x^2 - 48x + 52$.

Find the intervals where the functions is increasing, decreasing, concave up, concave down. Find critical numbers, points of inflection and any extrema.

Include your *Mathematica* notebook.

Clearly show your analysis and state your results below: