Section 13.4: The Cross Product

We saw that the dot product of two vectors produces a number. Now we define a product, called the cross product, of two vectors that produces another vector.

Suppose $\vec{v}$ and $\vec{w}$ are in the xy-plane as shown below:

We would like the vector $\vec{v} \times \vec{w}$ to be perpendicular to $\vec{v}$ and $\vec{w}$. Furthermore, we would like to magnitude of $\vec{v} \times \vec{w}$ to be equal to the area of the parallelogram formed by $\vec{v}$ and $\vec{w}$. What is the area of this parallelogram?

Which way does $\vec{v} \times \vec{w}$ point? See book's right-hand rule on page 673.

Cross product

Example: Let $\vec{v} = \vec{i}$ and $\vec{w} = \vec{i} + \vec{j}$. Find $\vec{v} \times \vec{w}$.

Now we can define $\vec{v} \times \vec{w}$ for any two vectors in 3 dimensions.

Geometric definition:

$\vec{v} \times \vec{w} = (\text{area of parallelogram})\vec{n} =$

where

Algebraic definition:

If $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$ and $\vec{w} = w_1\vec{i} + w_2\vec{j} + w_3\vec{k}$, then

$\vec{v} \times \vec{w} =$

Note: If $\vec{v}$ and $\vec{w}$ are parallel, there will be no parallelogram. In this case,

Example 3 (p. 675): Compute $\vec{v} \times \vec{w}$ where $\vec{v} = 2\vec{i} + \vec{j} - 2\vec{k}$ and $\vec{w} = 3\vec{i} + \vec{k}$.

Since the cross product helps us find a vector normal to other vectors, it can also help us find the equations of planes.

Example 4 (p. 676): Find the equation of the plane passing through the points $(1,3,0)$, $(3,4,-3)$, and $(3,6,2)$.

By using both the geometric and algebraic definitions of the cross product, we can find the area of the parallelogram determined by any to vectors in 3 dimensions.

Example 5 (p. 677): Find the area of the parallelogram determined by $\vec{v} = 2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{w} = \vec{i} + 3\vec{j} + 2\vec{k}$.

It turns out that the cross product and the dot product can be used to find the volume of a parallelepiped. If you're curious, see page 677.