

Calc 3 - Project 1

How to Build a Pyramid

Suppose you would like to build the four walls of a right pyramid with height h and square base with sides b . The walls will be made from sheets of material (say particle board) of a given thickness. Determine the exact dimensions of the four walls. In particular, determine the miter angles needed to have the four sides fit together properly.

Miter Angles

An angle cut along the edge of a sheet of material is called a miter angle. Viewing the edge of the sheet along the x -axis, the miter angle is the angle γ shown in Figure 1. For example, a typical wooden board purchased from a lumber yard has miter angle of 0° , and two boards that are to be joined at right angles would each have a miter angle of 45° along their common edge.



Figure 1: Miter Angles

Getting started:

Start with four isosceles triangles with base b and height $H = \sqrt{\left(\frac{b}{2}\right)^2 + h^2}$.

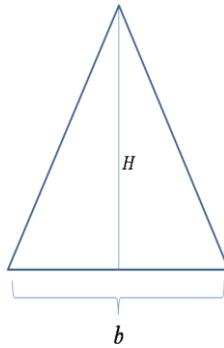


Figure 2: The walls of the pyramid

Note that h represents the height of the pyramid, whereas H represents the height (in the 2-dimensional sense) of the triangles that will form the walls of the pyramid. First find the miter angle for the bottom of the walls.

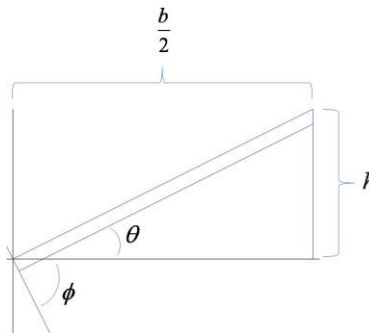


Figure 3: Miter angles for bottom of walls (side view)

To tackle the rest of the problem, view the pyramid in 3-dimensional space with its base in the xy -plane, the corners of the base at the points $\left(\pm\frac{b}{2}, \pm\frac{b}{2}, 0\right)$, and its vertex on the positive z -axis, then start thinking about planes!

Check point ☺

For a pyramid of base 10 and height $h=12$, the miter angle along the bottom of each wall is approximately 23° , and the miter angle along the sides of each wall is approximately 41° .

Bonus points problems

1. (1 point) Design a tetrahedron
 - a. Find the dimensions and miter angles for the walls of a right tetrahedron that has height h and whose base is an equilateral triangle with side s .
 - b. Find the miter angle for a regular tetrahedron.

2. (1 point) A construction crew is remodeling the second floor of an office building and will build a chute to guide debris into a truck below. The chute will have four walls, a large square opening at the top (say with side b_1), a smaller square opening at the bottom (say with side b_2), and length ℓ . Determine the dimensions of the walls of the chute and the miter angles for the sides.

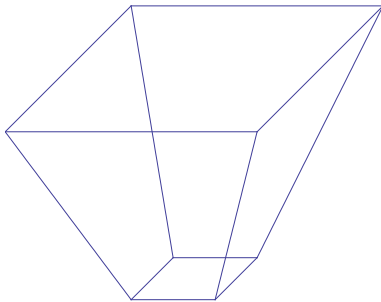


Figure 4: The chute

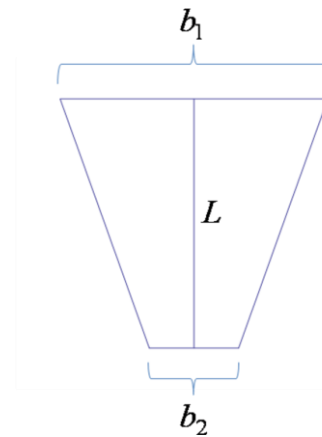


Figure 5: The walls of the chute