

MT 2800 Calculus III
WS 12.6: Limits and Continuity

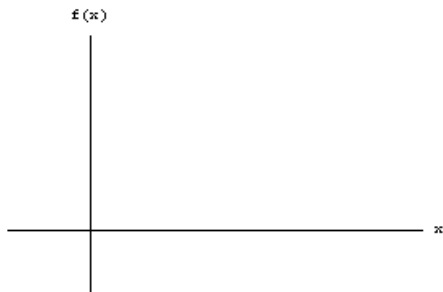
Limits and Continuity in 2D:

1. Limits in 2D:

$$\lim_{x \rightarrow a} f(x) = L < \infty \quad \Leftrightarrow$$

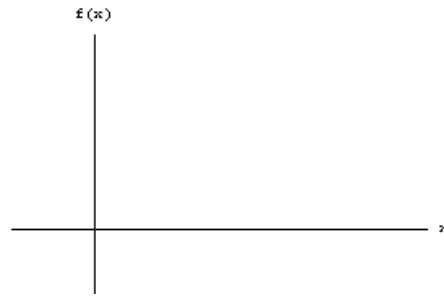
Less Rigorous:

More Rigorous:



Other Possibilities:

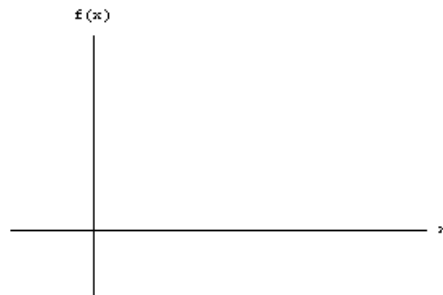
$$\lim_{x \rightarrow a} f(x) = \infty \quad \Leftrightarrow$$



Other Possibilities – continued:

$$\lim_{x \rightarrow a} f(x) \text{ DNE} \quad \Leftrightarrow$$

1)



2)

$$f(x) = \sin\left(\frac{1}{x}\right)$$

See Plot in Demo

2. Continuity in 2D:

- *Definition:* $f(x)$ is **continuous** at $x = a$ if $\lim_{x \rightarrow a} f(x) = f(a)$.

➤ Equivalently:

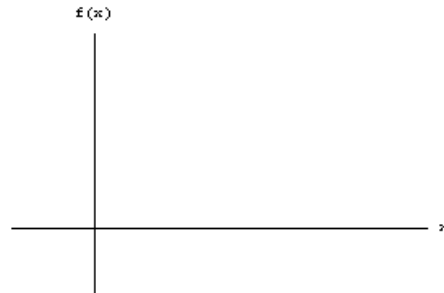
- 1.
- 2.
- 3.

- *Definition:* A function $f(x)$ is **continuous** (on the real numbers) if $f(x)$ is continuous at x for all real numbers, x .

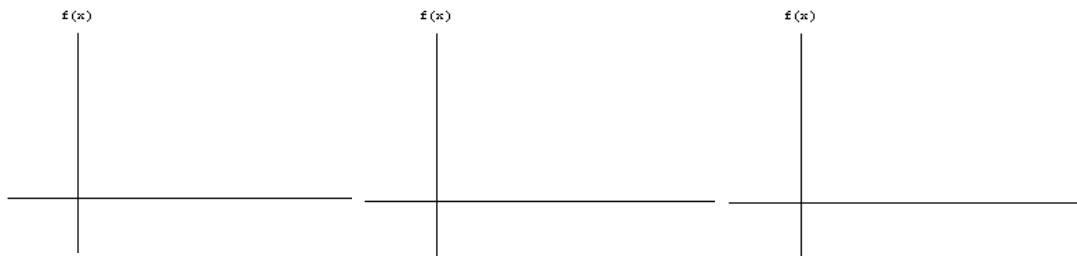
- *Layman's Interpretation:*

- *Examples:*

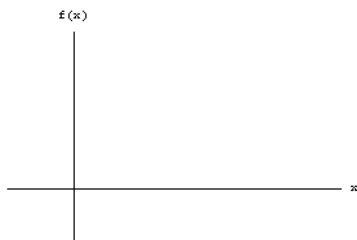
1) Continuous Everywhere:



2) Single Discontinuity:



3) Multiple Discontinuities:



Limits and Continuity in 3D:

1. Limits in 3D:

$$\lim_{(x,y) \rightarrow (a,b)} f(x,y) = L < \infty \quad \Leftrightarrow$$

2. Continuity in 3D:

- *Definition:* $f(x,y)$ is **continuous** at $(x,y) = (a,b)$ if $\lim_{(x,y) \rightarrow (a,b)} f(x,y) = f(a,b)$.

➤ Equivalently:

i.

ii.

iii.

- *Definition:* A function $f(x,y)$ is **continuous** (on a region A) if $f(x,y)$ is continuous at each point $(x,y) \in A$.

- *Layman's Interpretation:*

- *Examples:*

➤ Which of the following functions are continuous at (0,0), and which are not?

1. $f(x, y) = \frac{5x^2y+3}{1+x^2+y^2}$

○ Graphical: See Demo

○ Analytical:

2. $f(x, y) = \frac{1}{x^2 + y^2}$

○ Graphical: See Demo

○ Analytical:

$$3. \quad f(x, y) = \begin{cases} \frac{x^2 y}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

○ Graphical: See Demo

○ Analytical:

$$4. \quad f(x, y) = \left(\frac{x^2 - y^2}{x^2 + y^2} \right)^2$$

○ Graphical: See Demo

○ Analytical: