MT 2800 – Calculus 3
Worksheet 12.2
Simple Surfaces

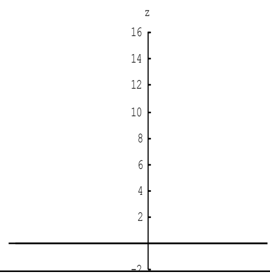
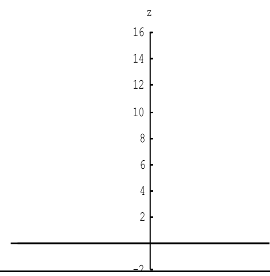
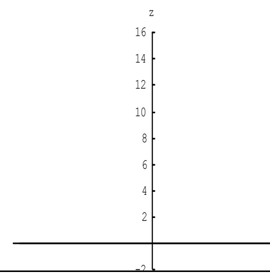
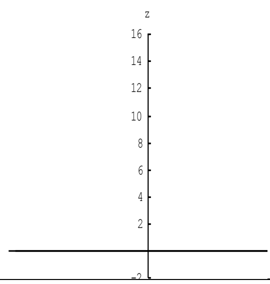
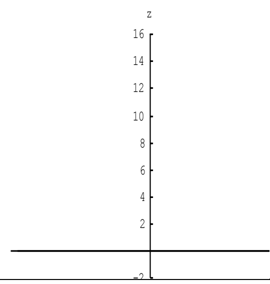
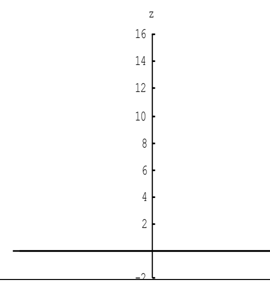
I. <u>Quadric Surfaces</u>:

1. The surface obtained by graphing the equation

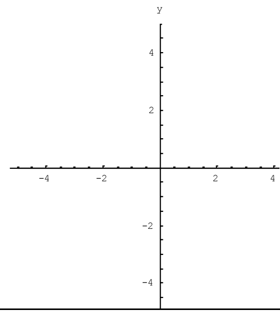
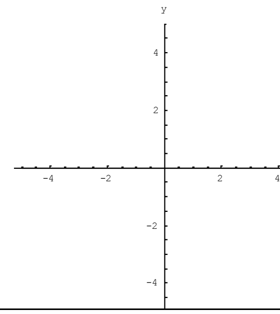
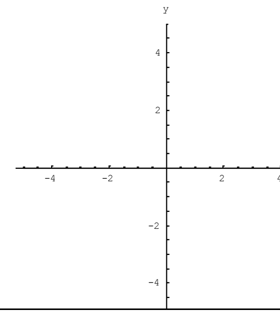
$$z = x^2 + y^2$$

is called a **circular paraboloid**.

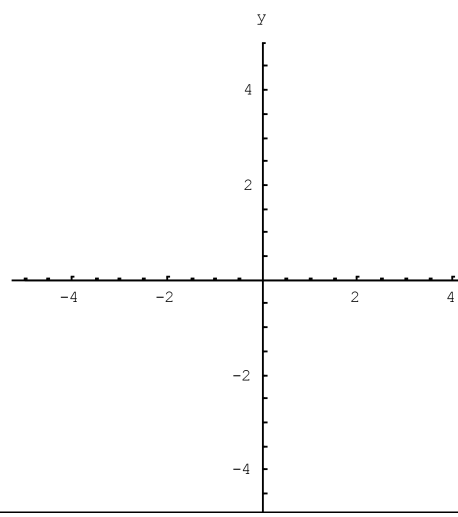
a. *Vertical Slicing*: Graph the curves obtained by intersecting this surface with the vertical planes: $y = -2,\ y = 0,\ y = 3,\ x = -2,\ x = 0,$ and $x = 3$:

Slice in the plane $y = -2$	Slice in the plane $y = 0$	Slice in the plane $y = 3$
		
Slice in the plane $x = -2$	Slice in the plane $x = 0$	Slice in the plane $x = 3$
		

b. *Horizontal Slicing*: Graph the curves obtained by intersecting this surface with the horizontal planes: $z=0$, $z=4$, $z=9$

Slice in the plane $z=0$	Slice in the plane $z=4$	Slice in the plane $z=9$
		

Now, drop all three horizontal slices graphed above into the xy - plane and create a **contour plot** for the surface $z=x^2+y^2$:

Contour Plot for $z = x^2 + y^2$


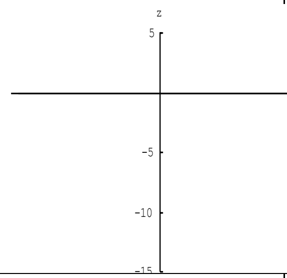
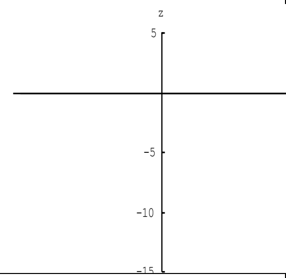
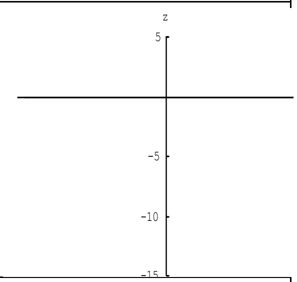
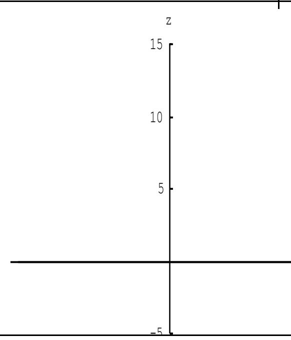
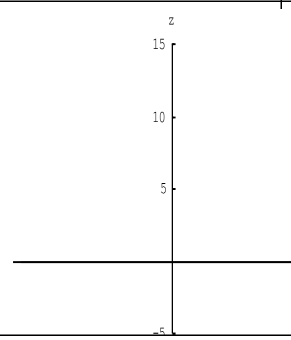
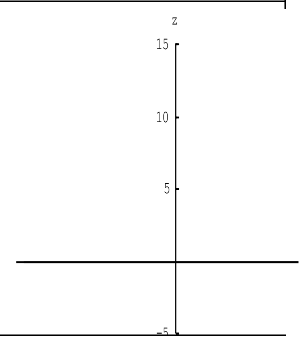
c. Given the visual information gathered by slicing your surface vertically and horizontally, describe and sketch the 3D surface (circular paraboloid) as best you can. Explain the relevance of the name "circular paraboloid".

2. The surface obtained by graphing the equation

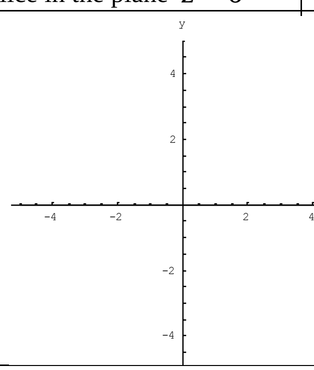
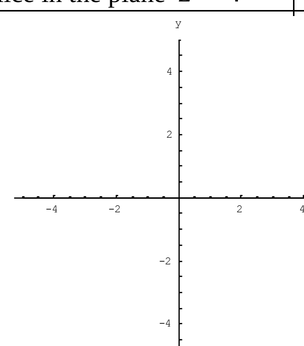
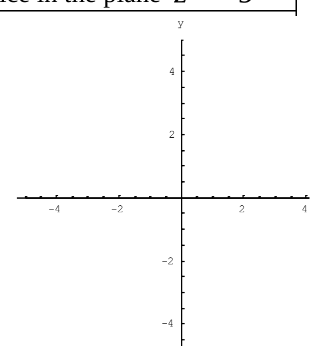
$$z = x^2 - y^2$$

is called a **hyperbolic paraboloid** (abbreviated by **hyperboloid**).

a. *Vertical Slicing*: Graph the curves obtained by intersecting this surface with the vertical planes: $y = -2,\ y = 0,\ y = 3,\ x = -2,\ x = 0,$ and $x = 3$:

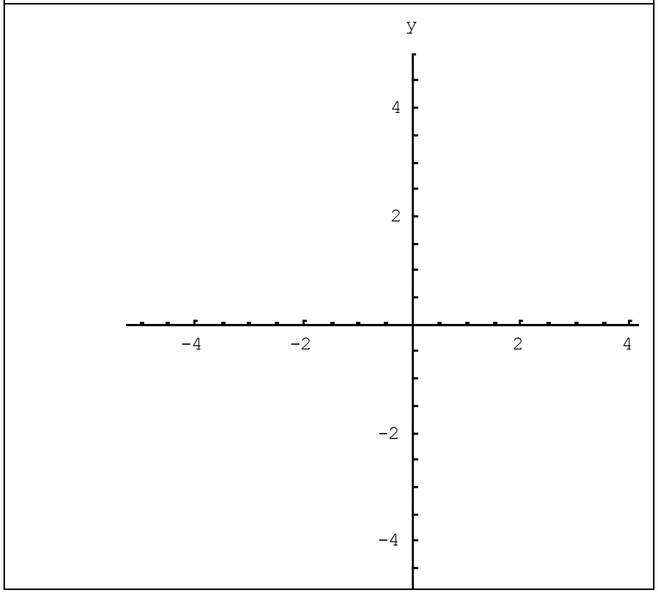
Slice in the plane $y = -2$	Slice in the plane $y = 0$	Slice in the plane $y = 3$
		
Slice in the plane $x = -2$	**Slice in the plane $x = 0$**	**Slice in the plane $x = 3$**
		

b. *Horizontal Slicing*: Graph the curves obtained by intersecting this surface with the horizontal planes: $z = 0,\ z = 4,\ z = -9$:

Slice in the plane $z = 0$	Slice in the plane $z = 4$	Slice in the plane $z = -9$
		

Now, drop all three horizontal slices graphed above into the xy - plane and create a **contour plot** for the surface $z = x^2 - y^2$:

Contour Plot for $z = x^2 - y^2$



c. Given the visual information gathered by slicing your surface vertically and horizontally, describe and sketch the 3D surface (hyperbolic paraboloid) as best you can. Explain the relevance of the name “hyperbolic paraboloid”.

II. Cylinders:

1. Any surface described by an equation in Cartesian coordinates with one coordinate missing is called a **cylinder**.

a. Sketch the surface in space described by the equation $x^2 + y^2 = 4$. Hint: Consider what this graph looks like in the xy-plane and realize that since the variable z is missing, it can take on any value.

b. Sketch the surface in space described by the equation $z = \cos x$.

c. Sketch the surface in space described by the equation $z = y^2$.