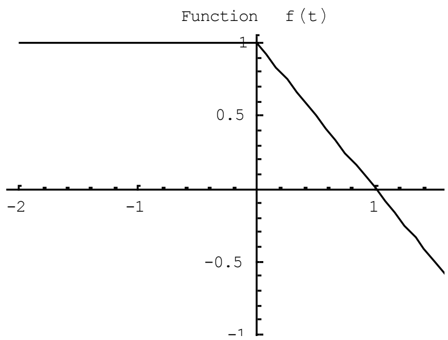
MT 1810 Calculus II
Worksheet: Sections 6.1 – 6.3
Review of Antiderivatives and Differential Equations

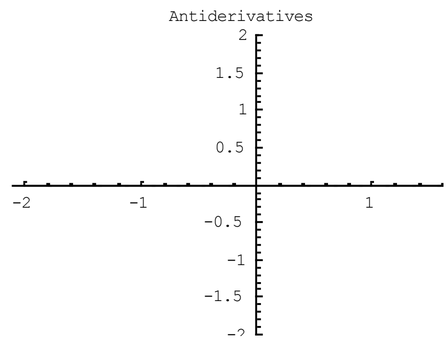
Name: ___________________________________

<u>Finding Antiderivatives Graphically and Numerically (Section 6.1):</u>

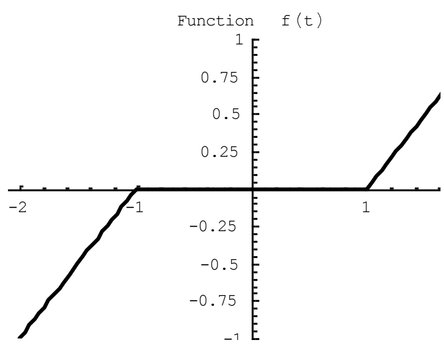
1. For each of the following, a graph of $f(t)$ is given. For each graph, sketch graphs of two different antiderivatives.

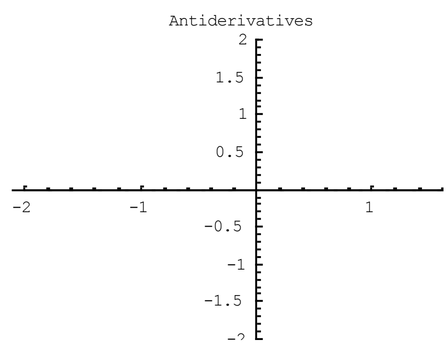
   a.





   b.

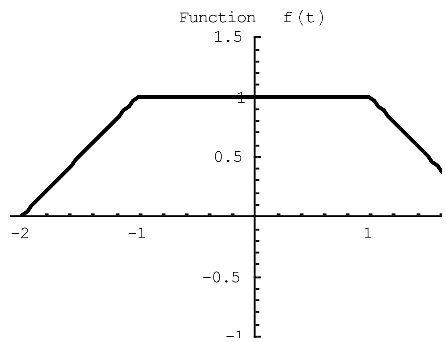




2. We can use the Fundamental Theorem of Calculus (FTC) to compute values of an antiderivative (see page 263 in the text). Suppose $F'(t) = f(t)$. We can rewrite the FTC as follows:

$$F(b) = F(a) + \int_a^b f(t)dt$$

Thus if we know the value of $F(a)$ and we have a formula or a graph for $f(t)$, we can compute the value of $F(b)$. Consider the graph of $f(t)$ given below:



a. Suppose $F(-2) = 0$. Compute $F(0)$.

b. Suppose $F(-1) = -2$. Compute $F(2)$.

3. Suppose you know that $f(t)$ is a decreasing function, and you are given some values in the chart below. Estimate the corresponding values of the antiderivative. Hint: Think area.

t	0	2	4	6	8
$f(t)$	10	6	4	3	2.5
$F(t)$	0				

Finding Antiderivatives Analytically (Section 6.2):

4. Find the following antiderivatives.

a. $\int \left(\sqrt{t} + \frac{1}{\sqrt{t}} \right) dt =$

b. $\int 5^x - x^5 \; dx =$

c. $\int \left(2 - \frac{1}{x} + e^{3x} \right) dx =$

d. $\int 5 \sin(-2t)\, dt =$

5. Use FTC to compute the following definite integrals.

a. $\int_1^4 \frac{1}{x^2}\, dx =$

b. $\int_0^1 (3x+3)^2\, dx =$

c. $\int_0^{\frac{\pi}{4}} \cos(2t)\, dt =$

Differential Equations (Section 6.3):

6. Is the function $y = e^{2x} + x^2 - 3$ a solution to the differential equation $y' = 2y + 6$? Show your work to justify your answer.

7. Find the general solution to the following differential equations.

a. $\dfrac{dy}{dx} = x - \sin x$

b. $\dfrac{dy}{dt} = 3 - e^{2t}$

8. Solve the following initial value problems (IVPs).

a. $\dfrac{dy}{dx} = 6x - \dfrac{1}{x}$, $y(1) = 0$.

b. $\dfrac{dy}{dx} = 5e^{x} + 2$, $y(0) = 1$.