Continuous functions

Def 21.1: f is continuous at c.
Let $f : D \to \mathbb{R}$ and let $c \in D$. We say that f is continuous at c, if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - f(c)| < \varepsilon$ whenever $x \in D$ and $0 < |x - c| < \delta$.

Thm 21.2 Let $f : D \to \mathbb{R}$ and let $c \in D$. Then TFAE:
(a) f is continuous at c.
(b) If $\{x_n\}$ in D is a sequence such that $\{x_n\}$ converges to c, then $\lim_{n\to\infty} f(x_n) = f(c)$.
(c) For every neighborhood V of $f(c)$ there exists a neighborhood U of c such that $f(U \cap D) \subseteq V$.
Furthermore, if c is an accumulation point of D, then the above are equivalent to:
(d) f has a limit at c and $\lim_{x\to c} f(x) = f(c)$.

Thm 21.6 Let $f : D \to \mathbb{R}$ and $c \in D$. Then f is discontinuous at c iff there exists a sequence $\{x_n\}$ in D such that $\{x_n\}$ converges to c but the sequence $\{f(x_n)\}$ does not converge to $f(c)$.

Thm 21.10 Let $f : D \to \mathbb{R}$ and $g : D \to \mathbb{R}$. be continuous and let $c \in D$. Suppose that f and g are continuous at c.Then:
(a) $f + g$ and fg are continuous at c and
(b) f/g is continuous at c if $g(c) \neq 0$.

Thm 21.12 Let $f : D \to \mathbb{R}$, $g : E \to \mathbb{R}$ be functions such that $f(D) \subseteq E$. if f is continuous at a point $c \in D$ and g is continuous at $f(c)$, then the composition $g \circ f : D \to \mathbb{R}$ is continuous at c.

Thm 21.14 A function $f : D \to \mathbb{R}$ is continuous on D iff for every open set G in $\mathbb{R}$ there exists an open set H in $\mathbb{R}$ such that $H \cap D = f^{-1}(G)$.

Corollary 21.15 A function $f : \mathbb{R} \to \mathbb{R}$ is continuous iff $f^{-1}(G)$ is open in $\mathbb{R}$ whenever G is open in $\mathbb{R}$.

HW: 21.1, 21.3, 21.4, 21.5, 21.6 (b, f), 21.16, 21.19.