Monotonic Sequences and Cauchy Sequences

Def 18.1: Increasing, decreasing, monotone
A sequence $\{s_n\}$ of real numbers is increasing if $s_n \leq s_{n+1}$, for all $n \in \mathbb{N}$.
A sequence $\{s_n\}$ of real numbers is decreasing if $s_n \geq s_{n+1}$, for all $n \in \mathbb{N}$.
A sequence $\{s_n\}$ of real numbers is monotone is it is either increasing or decreasing.

Thm 18.3 (Monotone Convergence Theorem)
A monotone sequence is convergent iff it is bounded.

Thm. 18.8
(a) If $\{s_n\}$ is an unbounded increasing sequence, then $\lim_{n\to\infty} s_n = +\infty$.
(b) If $\{s_n\}$ is an unbounded decreasing sequence, then $\lim_{n\to\infty} s_n = -\infty$

Def 18.9 Cauchy Sequence
A sequence $\{s_n\}$ of real numbers is said to be a Cauchy sequence if for each $\varepsilon > 0$ there exists a number N such that $m, n > N$ implies that $|s_n - s_m| < \varepsilon$.

Lemma 18.10 Every convergent sequence is Cauchy.

Lemma 18.11 Every Cauchy sequence is bounded.

Thm. 18.12 (Cauchy Convergence Criterion) A sequence of real numbers is convergent iff it is a Cauchy sequence.

HW: 18.1, 18.2, 18.3 (a,b,d), 18.5, 18.10, 18.11, 18.13.