Convergence

Def Sequence
A sequence $\{x_n\}_{n=1}^{\infty}$ in $\mathbb{R}$ is a function $f : \mathbb{N} \to \mathbb{R}$.

Def 16.2 a: Convergence
A sequence $\{x_n\}_{n=1}^{\infty}$ converges to $x_0 \in \mathbb{R}$ if for all $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that $|x_0 - x_n| < \varepsilon$ whenever $n \geq N$.

Def 16.2 b: Limit of a sequence
If $\{x_n\}_{n=1}^{\infty}$ converges to $x_0 \in \mathbb{R}$ then we write $\lim_{n\to\infty} x_n = x_0$, $\lim x_n = x_0$ or $x_n \to x_0$.

Def 16.2 c: Divergent sequence
If a sequence does not converge to a real number, it is said to diverge.

Theorem 16.8: Let $\{x_n\}_{n=1}^{\infty}$ and $\{a_n\}_{n=1}^{\infty}$ be sequences of real numbers and let $x_0 \in \mathbb{R}$. If for some $k > 0$ and some $m \in \mathbb{N}$ we have $|x_n - x_0| \leq k|a_n|$, for all $n > m$ and if $\lim_{n\to\infty} a_n = 0$, then it follows that $\lim_{n\to\infty} x_n = x_0$

Theorem 16.13: Every convergent sequence is bounded.

Theorem 16.14: Converging sequences have unique limits.

HW: 16.1, 16.2, 16.4, 16.6 (a,c), 16.7 (a, b, c, d), 16.8, 16.10.