2.8 Cardinality (a glimpse)

Def 8.1: Equinumerous sets
Two sets A and B are called equinumerous, denotes $A\tilde{}B$, if there is a bijective function from A onto B.

Def 8.3: Finite and infinite sets
A set A is said to be finite if $A = \varnothing$ or there exists $n \in \mathbb{N}$ and a bijection $f : \{1, 2, ..., n\} \to S$. If A is not finite it is said to be infinite.

Def 8.4: Cardinal number, transfinite set
The cardinal number of $I_n = \{1, 2, ..., n\}$ is n, and if $A\tilde{}I_n$, we say that A has n elements. The cardinal number of $\varnothing$ is 0. if the cardinal number of a set is not finite, then it is called transfinite.

Def 8.6: denumerable, countable, uncountable, $\aleph_0$
A set A is said to be denumerable if there exists a bijection $f : \mathbb{N} \to A$.
If a set is finite or denumerable it is called countable.
If a set is not countable then it is called uncountable.
The cardinal number of a denumerable set is denoted by $\aleph_0$.

Theorem 8.9: Any subset of a countable set is countable.

Theorem 8.10: Let S be a nonempty set. TFAE:
(a) S is countable
(b) There exists an injection $f : S \to \mathbb{N}$
(c) There exists a surjection $g : \mathbb{N} \to S$

Def. 8.16 Power set
Given any set S, let $\mathcal{P}(S)$ denote the collection of all subsets of S. The set $\mathcal{P}(S)$ is called the power set of S.

Continuum Hypothesis :$c = 2^{\aleph_0}$

HW. 2.8:#1.