2.7 Functions

Def 7.1: Function
Let A and B be sets. A function between A and B is a nonempty relation $f \subseteq A \times B$ s.t. if $(a, b) \in f$ and $(a, b') \in f$ then $b = b'$.

Def 7.1.a: Domain of a function
$domf = \{a \in A : \exists b \in B \ s.t. \ (a, b) \in f\}$

Def 7.1.b: Range of a function
$rngf = \{b \in B : \exists a \in A \ s.t. \ (a, b) \in f\}$

Note:
If the domain of f is the whole set A, then f is a function from A into B and we write $f : A \to B$.

Def 7.4: Surjective (onto) function
A function $f : A \to B$ is surjective (onto) if $B = rgnf$.

Def 7.5: Injective (1-1) function
A function $f : A \to B$ is injective (1-1) if for all a and a' in A, $f(a) = f(a')$ implies that $a = a'$.

Def 7.6: Bijective function
A function $f : A \to B$ is bijective if it is both surjective and injective.

Note: Let $f : A \to B$, $C \subseteq A$, $D \subseteq B$.

i) The image of C (in B) under f is $f(C) := \{f(x) : x \in C\}$.
ii) The preimage of D under f is $f^{-1}(D) = \{x \in A : f(x) \in D\}$.

Thm. 7.15 and 7.17: Suppose that $f : A \to B$. Let C, C_1 and C_2 be subsets of A and let D, D_1 and D_2 be subsets of B. Then the following hold:
(a) $C \subseteq f^{-1}[f(C)]$, equality holds when f is injective.
(b) $f[f^{-1}(D)] \subseteq D$, equality holds when f is surjective.
(c) $f(C_1 \cap C_2) \subseteq f(C_1) \cap f(C_2)$, equality holds when f is injective.
(d) $f(C_1 \cup C_2) \subseteq f(C_1) \cup f(C_2)$
(e) $f^{-1}(D_1 \cap D_2) \subseteq f^{-1}(D_1) \cap f^{-1}(D_2)$
(f) $f^{-1}(D_1 \cup D_2) \subseteq f^{-1}(D_1) \cup f^{-1}(D_2)$
(g) $f^{-1}(B \backslash D) = A \backslash f^{-1}(D)$

Def. Composition of functions
Let $f : A \to B$ and $g : B \to C$ be functions. The correspondence between $a \in A$ and $g(f(a)) \in C$ is called the composition function of f and g, denoted by $g \circ f$. $(g \circ f)(a) = g(f(a))$ for all $a \in A$.

Def 7.22 Inverse function
Let $f : A \to B$ be bijective. The inverse function of f is the function f^{-1}. $f^{-1} = \{(y, x) \in B \times A : (x, y) \in f\}$.

Recommended homework for section 2.7: 7.1, 7.3, 7.4, 7.6, 7.7 (b,c,f,g), 7.13, 7.14

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