Compact sets

Def 14.1 a:Cover
Let $E \subseteq \mathbb{R}$. Let $\{A_\alpha\}_{\alpha\in I}$ be a collection of subsets of $\mathbb{R}$. We say that $\{A_\alpha\}_{\alpha\in I}$ covers E (is a cover of E), if $E \subseteq \bigcup_{\alpha\in I} A_\alpha$.

Def 14.1 b:Finite subcover
Let $E \subseteq \mathbb{R}$. Let $\{A_\alpha\}_{\alpha\in I}$ be a cover of E. We say that $\{A_\alpha\}_{\alpha\in I}$ has a finite subcover of E if there exists a finite subset J of I such that the collection $\{A_\alpha\}_{\alpha\in J}$ covers E.

Def 14.1 c: Open cover
Let $E \subseteq \mathbb{R}$ and let any $\{A_\alpha\}_{\alpha\in I}$ be a cover of E..We say that $\{A_\alpha\}_{\alpha\in I}$ is an open cover of E, if every A_α is open in $\mathbb{R}$.

Def 14.1 d: Compact set
A subset $E \subseteq \mathbb{R}$ is said to be compact, if every open cover of E has a finite subcover.

Theorem: Compact subsets of $\mathbb{R}$ are closed.
The converse is not true in general. Ex. $\mathbb{R}$.

Lemma 14.4: Is E is a nonempty, closed and bounded subset of $\mathbb{R}$ then E has a maximum and a minimum.

Theorem: Closed subsets of compact sets are themselves compact.
Remark: Not every subset of a compact set is compact. Ex. [0,1] is compact yet (0,10 is not compact.

Theorem 14.5 (Heine-Borel) A subset $E \subseteq \mathbb{R}$ is compact iff E is closed and bounded.

Theorem 14.5 (Bolzano-Weierstrass) If a bounded subset $E \subseteq \mathbb{R}$ contains infinitely many points, then there exists at least one point in $\mathbb{R}$ that is an accumulation point of E.

Theorem 14.7: Let $\mathcal{F} = \{K_\alpha\}_{\alpha\in I}$ be a family of compact subsets of $\mathbb{R}$. Suppose that the intersection of any finite subfamily of $\mathcal{F}$ is nonempty. Then $\bigcap_{\alpha\in I} K_\alpha \neq \emptyset$.
Corollary 14.8: (Nested intervals Theorem) Let $\mathcal{F} = \{K_n\}_{n\in\mathbb{N}}$ be a family of closed bounded intervals in $\mathbb{R}$ such that $A_{n+1} \subseteq A_n$ for all $n \in \mathbb{N}$. Then $\bigcap_{n=1}^{\infty} A_n \neq \emptyset$.
HW: 14.1, 14.3, 14.4 and prove that if E is a finite subset of $\mathbb{R}$, then E is compact.