Motivation ε-δ definition of limit

If f is defined on an open interval containing a real number a,

except possibly at a, and if L is a real number,

then we say that $L = \lim_{x \to a} f(x)$ if and only if,

for every $\varepsilon > 0$ there exists a $\delta > 0$ such that

whenever $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$.

Statements

Def 1. A **statement** is a declarative sentence that is either true or false, but is not both true and false.

Def 2. The **truth value** of a statement is the designation T (true) or F (false), one and only one of which is assignable to any given statement.

Statements: examples

a) There is a rabbit on the face of the moon.
b) $x^2 \; {}^3 = x^6$
c) 6 is a prime number
d) January 1st, 2008 was a Tuesday.
e) The millionth digit of the decimal expansion of π is 4.

The following are not statements:

a) Is a dog a mammal?
b) Good morning!
c) Every orehu is a prentwitzhu of Haagen-dazs.
d) 2+3i is less than 5+3i.
e) $x>5$.
f) This sentence is false.

Why?

Logical connectives

Def 3. Given statements p and q, we define three statements formed from p and q.

a) The **negation of p**, denoted ~p and read "not p", is true precisely when p is false.

b) The **conjunction of p and q**, denoted $p \wedge q$ and read "p and q", is true precisely when p and q are both true.

c) The **disjunction of p and q**, denoted $p \vee q$ and read "p or q", is true when one or the other or both of the statements p and q is (or are) true.

Practice

Determine under what truth conditions the statement form for the compound statement
"either $\int_{-\pi}^{\pi} \sin x dx \neq 0$ and $\frac{d}{dx} \; 2^{x} \; = x2^{x-1}$ or $\int_{-\pi}^{\pi} \sin x dx = 0$ and $\ln 6 = (\ln 2)(\ln 3)$"
is true.

Hint: Use a truth table!!! ☺

Def 4. A statement form that is true under all possible truth conditions for its components is called a **tautology**.

Def 5. A statement for that is false under all possible truth conditions for its components is called a **contradiction**.

Def 6. Two compound statement forms that have the same truth values as each other under all possible truth conditions for their components are said to be **logically equivalent**.

p	q	~p	~q	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T					
T	F					
F	T					
F	F					

Def 7. Given statements p and q, we define:

a) The statement **p implies q**, denoted $p \rightarrow q$, also read "if p, then q", is true except in the case where p is true and q is false. Such a statement is called a **conditional**; the component statements p and q are called the **premise or antecedent** and **conclusion or consequent**, respectively.

b) The statement **p if and only if q**, denoted $p \leftrightarrow q$, also written "p iff q", is true precisely in the cases where p and q are both true or p and q are both false. Such a statement is called a **biconditional**.

The following mean exactly the same thing

If p, then q.
p implies q.
p only if q
p is a sufficient condition for q.
q if p.
q provided that p.
q whenever p.
q is a necessary condition for p.

p	q	r	~p	$\sim p \wedge q$	$p \wedge r$	$(\sim p \wedge q) \vee (p \wedge r)$
T	T	T				
T	F	T				
F	T	T				
F	F	T				
T	T	F				
T	F	F				
F	T	F				
F	F	F				