MT 3800 – Intro to Abstract Math
Proof Templates

- Existence Proof : $\exists x \ni P(x)$.

Theorem: There exists <something> that satisfies <a given assertion>.

Proof: Consider <Put your candidate here>. <Now show that your candidate satisfies the assertion>. ☺

- Existence and Uniqueness Proof : $\exists$ one and only one $x \ni P(x)$.

Theorem: There exists one and only one <something> that satisfies <a given assertion>.
Proof: Consider <Put your candidate here>.

1. <show that your candidate satisfies the assertion>.
2. Let <something 1 and something 2> be two objects that satisfy the assertion. <Now show that something1 = something2 or argue to a contradiction>. ☺

- Direct Proof : $A \Rightarrow B$.

Theorem: If <state hypothesis> then <state conclusion>
Proof:
Suppose <Write the first sentence of the proof by restating the hypothesis.>

Therefore <Write the last sentence of the proof by restating the conclusion.> ☺

Note: The devil is in the details between the first and last sentence of the proof! On a separate sheet of paper, use definitions and previous theorems to move forward from the first sentence, and backward from the end of the sentence. Try to forge a link between the two halves of your argument.

- Proof by Contrapositive: $(A \Rightarrow B) \Leftrightarrow (\sim B \Rightarrow \sim A)$

Theorem: If <state hypothesis> then <state conclusion>
Proof:
Suppose <Write the first sentence of the proof by stating the negation of the conclusion.>

Therefore <Write the last sentence of the proof by stating the negation of the hypothesis.> ☺

Note: As in the case of the direct proof, you want to use a forward-backward approach to link the first and last lines together.

Revised 1/20/2009

- Proof by Contradiction: $(A \Rightarrow B) \Leftrightarrow [(A \wedge \sim B) \Rightarrow (P \wedge \sim P)]$.

Theorem: If <state hypothesis> then <state conclusion>
Proof:
Assume <the hypothesis> is true but <the conclusion> is false.

Include arguments until you reach <u>any</u> contradiction.
$\Rightarrow\Leftarrow$ ☺

Notes:
1. The contradiction may have nothing to do with the theorem being proved.
2. "$\Rightarrow\Leftarrow$" is an abbreviation for "We have reached a contradiction, therefore our assumption that the negation of the conclusion is false, so the conclusion must be true."

- Another type of Proof by Contradiction: $A \Leftrightarrow [\sim A \Rightarrow (P \wedge \sim P)]$.

Theorem: <State assertion>
Proof:
Assume <the assertion> is false.

Include arguments until you reach <u>any</u> contradiction.
$\Rightarrow\Leftarrow$ ☺

Notes:
3. The contradiction may have nothing to do with the theorem being proved.
4. "$\Rightarrow\Leftarrow$" is an abbreviation for "We have reached a contradiction, therefore our assumption that the negation of the assertion is false, so the assertion must be true."